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Study of the relationship between the V–I curve and the flux dynamics in superconductors

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Abstract
The equivalence of the experimental V–I characteristic ($V \propto I^m$) and the material E–j curve ($E \propto j^n$) was studied. We show numerically that only when the current I is larger than I_{hom}, the fully penetrating current, can the V–I curve be equivalent to the E–j one and thus be used to determine the material parameter n, whereas if $I < I_{hom}$, j is inhomogeneous and the V–I curve is not appropriate for use in probing the properties of the sample. The inhomogeneity of j can be checked by simply measuring the voltage relaxation curve at a given I. Furthermore, it is shown that I_{hom} varies with $\mathrm{d}I/\mathrm{d}t$ and n. The dependence of I_{hom} on $\mathrm{d}I/\mathrm{d}t$ indicates that current cannot go directly into a homogeneous region in practical transport measurements. Moreover, we argue that all of the V–I curves with different values of the flux-creep barrier exponent μ merge at large current. Since current is probably inhomogeneous in the small-current region, the V–I curve might not be appropriate for use in studying the properties of the sample in this region. Therefore, the V–I curve may not be appropriate for determining μ in the U–j relation at all sensitively.

1. Introduction

Investigation of the E–j characteristic curve is one of the basic ways of achieving an understanding of and probing vortex behaviour [1–5]. However, in practical measurements, only the V–I curve can be observed directly. In order to obtain the E–j curve, it is usually assumed that current distributes homogeneously in superconductors. Thus, the measured V–I curves can be directly changed to E–j ones. However, there are difficulties in explaining some experiments based on this assumption. For example, it is found that the V–I curve of Ag-sheathed Bi-2223 tape depends on the sweeping rate of the applied current ($\mathrm{d}I/\mathrm{d}t$) [5].

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It has also been reported that the resistance of $YBa_2Cu_3O_{7-\delta}$ film decays with time, i.e. the resistance relaxes [6]. It is noted that these experiments can be understood if one assumes that the current in the sample is spatially inhomogeneous [7, 8]. In fact, the local current density (j) may be several times higher than the average one in some cases. Such inhomogeneous current may be encountered more frequently in transport and field-sweeping measurements. For example, to circumvent the ohmic heating in conventional four-terminal measurements, a pulsed-current system with a large $\mathrm{d}I/\mathrm{d}t$ has been developed[4]. And in field-sweeping measurements, such as magnetization measurements and ac susceptibility experiments, the magnetic shielding current density depends on the sweeping rate of the applied field ($\mathrm{d}H/\mathrm{d}t$) and the frequency, respectively. How does the inhomogeneous current density in transport measurements affect the characteristic V–I curve? How does one obtain the material equation $E \propto j^n$ from an experimental V–I curve? One of the purposes of the present paper is finding answers to these questions.

It is generally accepted that at $j \to j_{c0}$ and low temperature, flux creep can be well described by the Anderson–Kim model [9, 10], whereas the vortex-glass model [11, 12] predicts that resistivity induced by flux creep decreases exponentially with j, indicating that a very small j is needed to clarify these models. It is not established that an experimental V–I curve at small I can still be transformed to an E–j curve directly.

It is well known that different flux-creep models are characterized by different flux motion barriers described by the U–j relationship, and that the U–j relation has a specific exponent μ. It is expected that different barriers will yield different V–I curves [13]. The other purpose of this work is to find out whether the V–I curve can be used to determine μ in a general U–j relation.

In practice, a long cylinder is a good approximation for a sample, and this shape is also frequently encountered in experiments. Therefore, numerical simulations as well as transport experiments on cylindrical superconductors are studied in this paper. By solving the electric dynamic equation together with the material equation numerically, we calculated the V–I curve and the underlying electric field and current-density evolution in transport measurements in different cases. The calculation was carried out by assuming the glass exponent μ to be 0 at first, and then showing that the results are independent of μ.

2. Numerical simulations

2.1. Basic equations

Let the sample be an infinitely long cylinder with radius R and the current be applied at a rate of $\mathrm{d}I/\mathrm{d}t$. Cylindrical coordinates (r, ϑ, z) are used. The current density and the electric field are directed along the z-axis (the cylinder axis), and the vortex density is directed along the ϑ-direction; these are denoted by $j(r, t)$, $E(r, t)$, $B(r, t)$, respectively. Therefore, the Maxwell equations can be written as

$$\frac{\partial B}{\partial t} = \frac{\partial E}{\partial r} \qquad \frac{1}{r}\frac{\partial}{\partial r}(rB) = \mu_0 j. \tag{1}$$

The electric field E induced by the flux creep is assumed to follow the power-law material equation [14–21]

$$E = E_0\left(\frac{j}{j_{c0}}\right)^n \tag{2}$$

[4] See, for examples, references [1–4] in reference [13].

where j_{c0} is the critical current density and E_0 is the electric field when $j = j_{c0}$. If a logarithmic barrier, $U = U_0 \ln j_{c0}/j$, is assumed [5], equation (2) is easy to obtain, and the exponent $n = U_0(T, H)/kT$ depends on the temperature T, magnetic field H, and pinning strength. Incidentally, for $n = 1$, the equation is reduced to Ohm's law, describing the normal state or flux flow. For large n, the equation describes the Bean critical state: $E = E_0$ for $j = j_{c0}$; $E = 0$ for $j = 0$. When $1 < n < \infty$, the equation describes nonlinear flux creep of the vortex glass.

Substituting the material equation into equation (1), we get the diffusion equation for the electric field:

$$\frac{\partial E}{\partial t} = \frac{nE_0^{1/n}}{\mu_0 j_{c0}} E^{1-1/n} \left(\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} \right). \tag{3}$$

2.2. The boundary and initial conditions

In V–I measurements, if current is applied at a rate $\mathrm{d}I/\mathrm{d}t$ and the current is inhomogeneous in the sample, one has to relate I and j in terms of an integral:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^R j\,2\pi r\,\mathrm{d}r = \frac{\mathrm{d}I}{\mathrm{d}t}.$$

Because

$$\mu_0 \frac{\mathrm{d}}{\mathrm{d}t} j = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial E}{\partial r}\right)$$

integrating this equation yields

$$2\pi r \left.\frac{\partial E}{\partial r}\right|_{r=R} - 2\pi r \left.\frac{\partial E}{\partial r}\right|_{r=0} = \mu_0 \frac{\mathrm{d}I}{\mathrm{d}t}. \tag{4}$$

In view of the central symmetry of the sample, one has

$$\left.\frac{\partial E}{\partial r}\right|_{r=0} = 0. \tag{5a}$$

By means of equation (5a), equation (4) is reduced to

$$\left.\frac{\partial E}{\partial r}\right|_{r=R} = \frac{\mu_0}{2\pi R}\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{5b}$$

Equations (5a) and (5b) are the boundary conditions on the V–I curve measurement where $\mathrm{d}I/\mathrm{d}t$ is known.

The initial condition is simple:

$$E(r, t)\big|_{t=0} = 0. \tag{6}$$

Thus, we can first calculate $E(r, t)$ at any time and position in space according to the above basic equation (3), the boundary conditions (equations (5a), (5b)), and the initial condition (equation (6)), and then $j(r, t)$ according to equation (2).

In order to calculate the V–I characteristic curve, which can be directly measured in transport experiments, we assume that

$$V = \langle E\rangle L = \frac{L}{\pi R^2}\int_0^R E\,2\pi r\,\mathrm{d}r$$

where L is the distance between the voltage contact points. As stated above,

$$I = \int_0^R j\,2\pi r\,\mathrm{d}r.$$

Hence, the V–I curve can be obtained by integrating with respect to E and j.

3. Results and discussion

Although both E and j were calculated, only j is shown here for simplicity. In fact, j is calculated according to the material equation (2).

Illustrated in figure 1(a) is a typical current-density profile of a cylindrical sample when the current is applied with dI/dt constant.

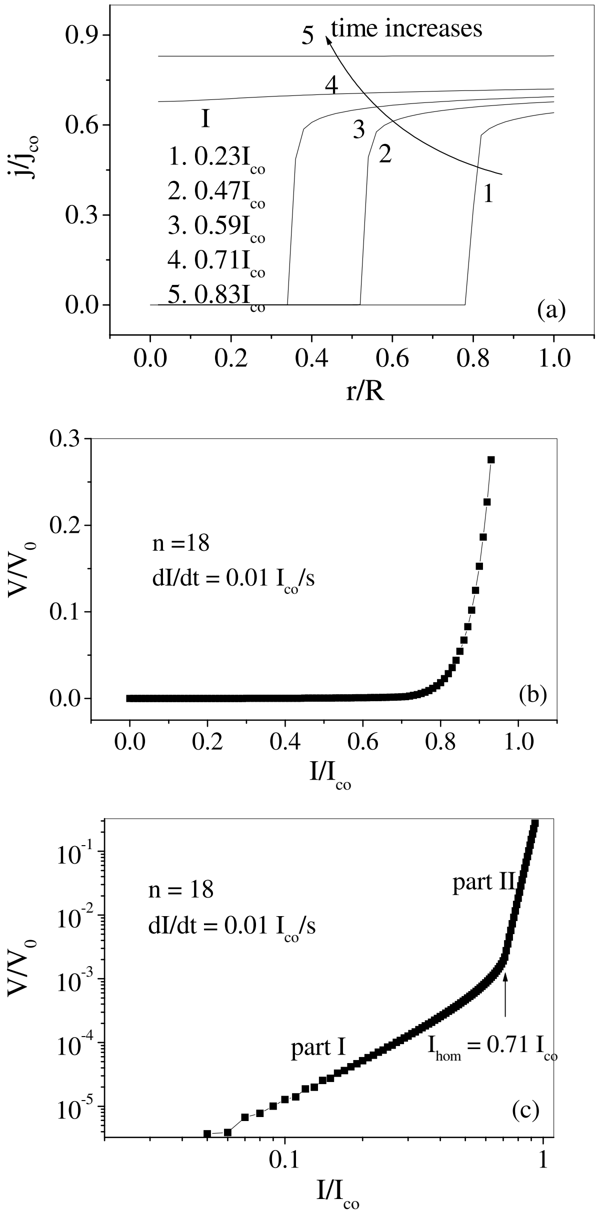


Figure 1. (a) Numerical space evolution of the current density during current sweeping at $n = 18$ and $dI/dt = 0.01 I_{c0}\ s^{-1}$. $r/R = 0$, 1 denote the centre and surface of the cylinder, respectively. The arrow indicates the direction of time. (b) The corresponding V–I curve. (c) The corresponding log–log V–I curve.

Evidently, j is spatially inhomogeneous when the current is small. Current flows near the surface at first, then expands towards the centre. After a period of time, j near the surface may be much larger than the average one, while j in the central part is almost zero, making the effective current-carrying area much smaller than the whole cross-section. For example, at time $t = 23$ s ($t = I/(\mathrm{d}I/\mathrm{d}t)$, $I = 0.23I_{c0}$) in the case of figure 1(a), the current density near the surface ($j \approx 0.64j_{c0}$) was about double the average one ($0.23j_{c0}$) whereas j in the central part is still too small to be detected. Nevertheless, as time increases (current increases), the current density becomes gradually homogeneous. When the current is large enough—say, $I = 0.71I_{c0}$ in the case of figure 1(a)—j is almost the same from point to point. The V–I curve corresponding to figure 1(a) is depicted in figure 1(b). If one attempts to fit the V–I curve to a power-law relation $V \propto I^m$, the exponent m equals the slope of the log–log V–I curve. For clarity, the log–log V–I curve is shown in figure 1(c). Evidently, m varies with current and is not always identical to n in the material equation. It is seen that the V–I curve can be divided into two main parts, which nearly meet at $I = 0.71I_{c0}$. Here we tentatively define this transition current I as I_{hom}, at which the transition from an inhomogeneous current density to a homogeneous one takes place. In the case of figure 1(c), $I_{hom} = 0.71I_{c0}$. For convenience, we call the region in the V–I curve where the current is smaller than I_{hom} part I, and the one where the current is larger than I_{hom} part II. In part I, current is inhomogeneous (see figure 1(a)) and $m \approx 2$, as in figure 1(c). At the moment we do not know the physical meaning of m. It is due to the inhomogeneous current density that the V–I curve in this region is not identical to the E–j curve. In view of this, it is not appropriate to check the vortex dynamic behaviour predicted by the vortex-glass model using the V–I curve in this region. In part II, where $m \approx 18$, the current fully penetrates the sample and is almost spatially homogeneous. The fact that $m \approx n$ indicates that the V–I curve in this region is equivalent to the E–j one, and can be used to determine n. At $I \approx I_{hom}$, m changes gradually from 2 to n.

As stated above, the current may be inhomogeneous in V–I measurements. How does one check whether the current distribution is uniform at a given I? In the following, it is shown that the simplest way is to measure the voltage (resistance) relaxation curve (V–t curve) at a given I. In a voltage relaxation experiment, the global current I is kept constant and the voltage relaxation results from the current diffusion from the local area to the whole sample. That is to say, if the voltage relaxes at a given I, the corresponding current distribution is inhomogeneous. This situation is quite different from the one encountered in magnetic relaxation experiments, where the current varies during the period of relaxation. On the basis of the above idea, the situation regarding the current distribution can be determined. For clarity, an example of a sweep–relaxation curve is shown in figure 2. Since the homogeneity of j is different during current sweeping as seen in figure 1(a), different degrees of relaxation are obtained at different values of I. In the inset, the variation of the degree of relaxation D with I is shown. Here we define $D = (V_{ini} - V_{res})/V_{ini}$, where V_{ini} is the initial relaxation voltage, V_{res} is the residual voltage. It is seen that $D \approx 0$ when $I > I_{hom}$, whereas D is large at $I < I_{hom}$. In the vicinity of I_{hom}, D changes substantially and quickly, indicating that I_{hom} is the current at which j changes from being inhomogeneous to being homogeneous. Therefore, as long as $D \neq 0$, the current is inhomogeneous.

To see whether the above result holds for different values of n, we calculated the V–I curves at several n-values; the results are shown in figure 3(a). Depicted in figure 3(b) are the corresponding double-logarithmic curves. As in the above, each of the curves ($n = 3, 5, 10, 30, 50$) in figure 3(b) can be divided into part I and part II. Only in part II are m and n nearly equal to each other. For clarity, the values of m in the different parts of figure 3(b) are listed in table 1. It is noted that I_{hom} varies with n. The dependence of I_{hom} on n is displayed in the inset of figure 3(b). It is seen that the larger n, the larger I_{hom}. For

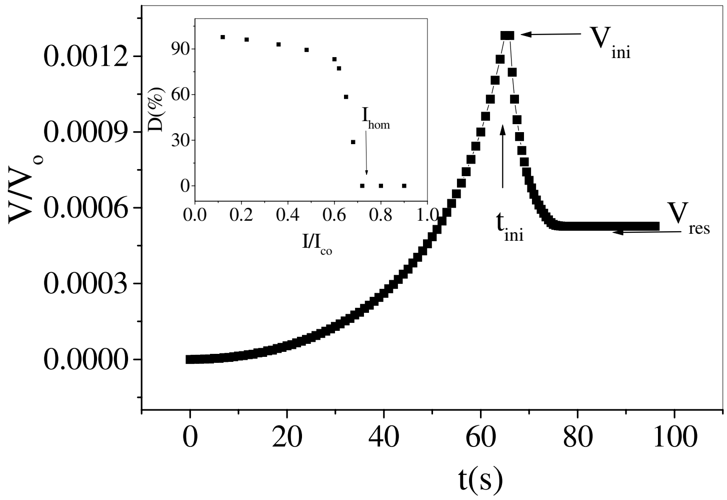


Figure 2. The sweep–relaxation curve at $n = 18$ and $dI/dt = 0.01I_{c0}$ s^{-1}. When $t < t_{ini}$, the current is swept, whereas when $t > t_{ini}$, the current is kept constant and the voltage relaxes. The inset shows the dependence of D on I, where $D = (V_{ini} - V_{res})/V_{ini}$ denotes the degree of relaxation, V_{ini} is the initial relaxation voltage, and V_{res} is the residual voltage.

Table 1.

n	Part I m	Part II m
3	1.82	2.995
5	1.9	4.996
10	2.1	9.99
30	2.12	29.6
50	2.13	49.7

example, $I_{hom} = 0.3I_{c0}$ at $n = 5$ (a typical magnitude for Ag-sheathed $Bi_2Sr_2Ca_2Cu_3O_{10}$ tape at zero magnetic field and liquid-nitrogen temperature), whereas $I_{hom} = 0.87I_{c0}$ at $n = 50$ (a typical magnitude for NbTi–Cu wire for a 1 T magnetic field and at 11 K). The corresponding current profile at $I = 0.25I_{c0}$ is shown in figure 3(c). It is obvious that the current is almost homogeneous at $n = 3$, whereas the current is strongly inhomogeneous at $n = 30$.

Now we show the dependence of I_{hom} on dI/dt in figure 4(a). The inset shows the I_{hom}–dI/dt relation more clearly. It is seen that I_{hom} shifts towards small current with decreasing dI/dt. However, the condition that $I_{hom} \to 0$ is impossible to achieve, because in the experiment, current cannot be applied at an infinitely low rate. Thus, no matter how slowly current is applied in the experimental limit, I_{hom} always has a finite value. That is to say, in practical measurements, current cannot go directly into the homogeneous region. In fact, the I_{hom}–dI/dt curve divides the plane into two regimes. Above the curve, the current is homogeneous, whereas below the curve, the current is inhomogeneous. The corresponding current profile at $I = 0.7I_{c0}$ is shown in figure 4(b). It is found that the more slowly the current is applied, the more homogeneous the current is.

All of the above results are based on the logarithmic U–j relation, which is equivalent to $\mu = 0$ in the following U–j relation:

$$U(j) = \left(\frac{U_0}{\mu}\right)\left[\left(\frac{j_{c0}}{j}\right)^{\mu} - 1\right]. \quad (7)$$

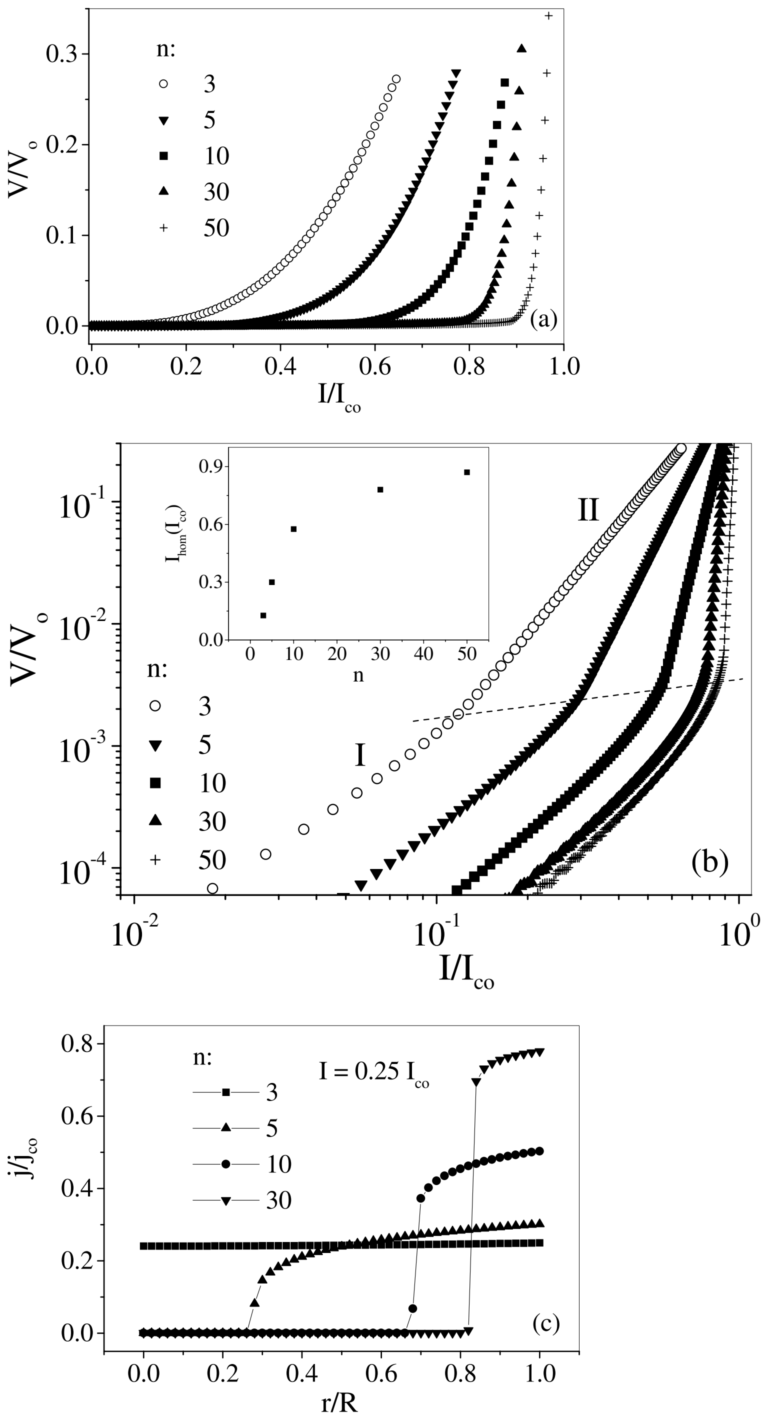
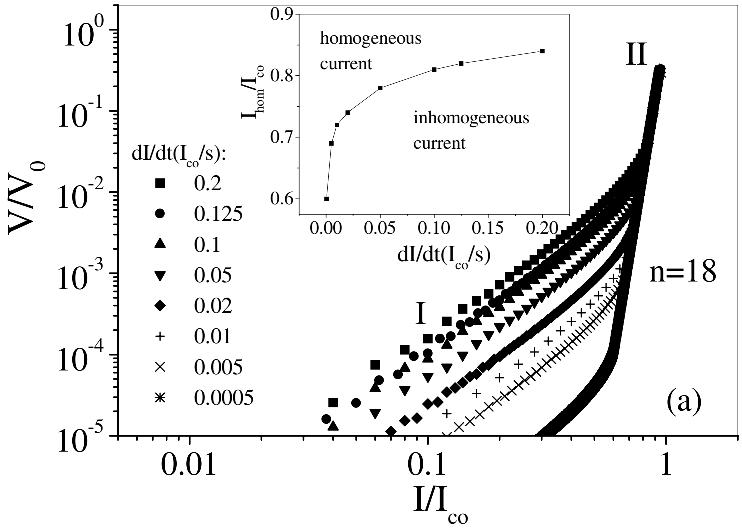


Figure 3. (a) The dependence of V–I curves on n at a constant $\mathrm{d}I/\mathrm{d}t$ ($=0.02I_{c0}$ s^{-1}). (b) The corresponding double-logarithmic V–I curve. The dashed line indicates the boundary between part I and part II. The inset shows the dependence of I_{hom} on n. (c) Current-density profiles at different n-values.



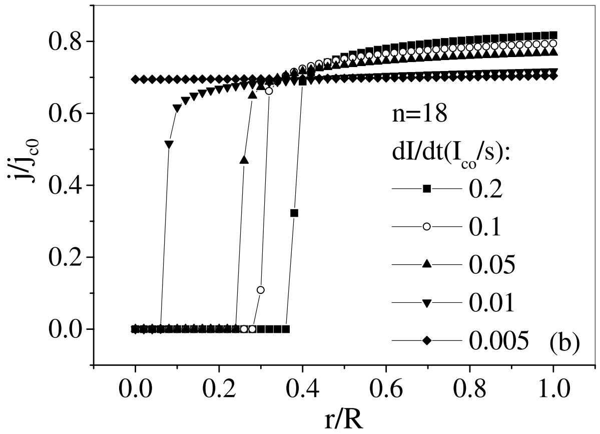


Figure 4. (a) The numerical double-logarithmic V–I curves for different values of $\mathrm{d}I/\mathrm{d}t$ at $n = 18$. The inset shows the dependence of I_{hom} on $\mathrm{d}I/\mathrm{d}t$. (b) The corresponding current-density profile at $I = 0.7I_{c0}$.

For $\mu = -1$, equation (7) reduces to the Anderson–Kim model

$$U(j) = U_0\left(1 - \frac{j}{j_{c0}}\right). \tag{8}$$

For $\mu \neq -1$, equation (7) describes the vortex-glass model.

In fact, equation (8), which is independent of μ, is a special case for $j \to j_{c0}$ of equation (7), which is dependent on μ. When $j \to j_{c0}$, equation (7) can be reduced to

$$U(j) = \left(\frac{U_0}{\mu}\right)\left[\left(\frac{j_{c0} - j + j}{j}\right)^{\mu} - 1\right] = \left(\frac{U_0}{\mu}\right)\left[\left(1 + \frac{\Delta j}{j}\right)^{\mu} - 1\right]$$
$$= \left(\frac{U_0}{\mu}\right)\left[1 + \mu\left(\frac{\Delta j}{j}\right) - 1\right] = U_0\left(\frac{\Delta j}{j}\right) \tag{9}$$

which shows that the flux barrier $U(j)$ is independent of μ when $j \to j_{c0}$. It is well known that the V–I curve is calculated by assuming a kind of U–j relation. The independence of

$U(j)$ from μ results in V–I curves for different values of μ that are the same, at least at large current. In fact, numerical E–j curves for the three cases of $\mu = 1, 0, -1$ which merge at large current densities have been reported in [22]. In other words, the V–I curve cannot be used to determine the glass exponent μ when the applied current is near to I_{c0} even when j is spatially homogeneous. Because current is probably inhomogeneous when I is small, the V–I curve might not be appropriate for use in probing the properties of the sample in this region. Therefore, one can see that not all parts of the V–I curves are appropriate to use to determine μ with any sensitivity.

4. Summary

We have studied the equivalence of the V–I characteristic curve measured in transport experiments and the E–j curve for the power law ($E \propto j^n$) describing the properties of a material numerically and experimentally. By solving the flux diffusion equation numerically, we calculated V–I curves and the underlying $E(r, t)$ and $j(r, t)$ curves. It was found that the numerical V–I curves can be fitted by the power law $V \propto I^m$ with different values of m, and that they each consist of two parts separated at a transition current I_{hom}. In the region where $I > I_{hom}$, j is homogeneous and $m \approx n$, indicating that the V–I curve is equivalent to the E–j one. Thus only in this region can the V–I curve be used to determine n. When $I \approx I_{hom}$, m increases gradually and finally equals n. In the region where $I < I_{hom}$, $m < n$ and j is inhomogeneous; the V–I curve cannot then be used to determine the intrinsic properties of a superconductor. The inhomogeneity of j can be checked by simply measuring the voltage relaxation curve at a given I. If the voltage relaxes at a constant I, the corresponding j is inhomogeneous. Furthermore, it is shown that I_{hom} varies with $\mathrm{d}I/\mathrm{d}t$ and n. The dependence of I_{hom} on $\mathrm{d}I/\mathrm{d}t$ indicates that current cannot go directly into a homogeneous region in practical transport measurements, for it cannot be applied at an infinitely low rate. Moreover, the dependence of the V–I curve on the glass exponent μ is also studied. It is found that the V–I curves for different values of μ are the same as each other, at least at large current. However, in the small-current region, current is probably inhomogeneous, resulting in the V–I curve being inappropriate for use in probing the properties of the sample. Therefore, the V–I curve might not be appropriate for determining μ in the U–j relation with any sensitivity.

Acknowledgments

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